Game Theory Approach to Solve Economic Dispatch Problem

Nezihe Yildiran and Emin Tacer

Abstract—**This paper presents a game theory application for economic dispatch problem. In this application, economic dispatch problem has been applied to 6 thermal stations in 14-bus, 380kV power system in Turkey, by taking the generator limits and total demand into account, but transmission losses have been neglected. To find the optimum dispatch strategy Nash equilibrium has been used with MATLAB. Instead of following any particular strategy, the entire system has been analyzed. At the end of this paper, it was shown; calculated system total cost is smaller than the total cost of genetic algorithm and Lagrange function applications.**

Index Terms—**Economic dispatch, game theory, Nash equilibrium, optimization.**

NOMENCLATURE

C_i: Cost of i-th generator [$/h].
α_i: Cost coefficient of i-th generator [$/h].
β_i: Cost coefficient of i-th generator [$/MWh].
γ_i: Cost coefficient of i-th generator [$/MW^{2}h].
P_i: Generated real power of i-th generator [MW].
n: Number of generating units.
C_T: Total cost of generators [$/h].
P_D: Total load demand [MW].
P_L: Transmission losses [MW].
$P_{i(\min)}$: Minimum generating limits for plant *i* [MW].
$P_{i(\max)}$: Maximum generating limits for plant *i* [MW].
GP_*i*: *i*-th generation plant.

I. INTRODUCTION

Economic dispatch is generally defined as the determining the operation points of the generation plants at the lowest cost to supply demand while considering generator limits and transmission line lengths.

Nowadays, energy demand is increasing because of growing population and increasing of the living standard level in the world. On the other hand, non-renewable energy sources are decreasing and renewable energy sources are getting popular with their discrete production areas. Furthermore, transmission line length, which is the distance between power generation plants and load centers, is changing. Otherwise, in an interconnected system, the power plants use different sources such as natural gas, water, and coal; and the prices of these sources are variable. Under normal operating conditions, the total generation capacity of the generation plants is more than the total load demand and losses. Thus, there are many options for scheduling power generation. Hence, today's economic dispatch problem is rapidly gaining importance, because energy is very precious now.

In summary, for any specified load condition economic dispatch determines the power output of each power plant which will minimize the overall cost of fuel needed to maintain the system total demand. Economic dispatch focuses upon coordinating the production costs at all power plants operating on the power system. So, energy should be used efficiently and rightly. Analyzing ED problem helps finding best scheduling solutions. The optimization of economic dispatch shows the economic value of the network operator. The economic dispatch is a relevant procedure in the operation of a power system. Because of the daily demand variation on the power system, the utility has to decide on the basis of economics which generators to start up, which to shut down, and in what order.

Over the past years, economic dispatch (ED) problem is studied and solved by using different mathematical or optimization methods in the literature. One of the literature applications is Hopfield neural network method, which is used by Yalcinoz and Short to solve the ED problem [1]. In the Yalcinoz's paper, smaller solution time and smaller operation costs according to classical optimization methods were founded. Generally, genetic algorithm has applied to solve the ED problem. Kurban and Basaran solved the ED problem with using Lagrange functions [2]. Dosoglu, Duman, and Ozturk used Kurban and Basaran's [2] system and applied genetic algorithm [3] to the same system. Afterward, Gaing have proposed particle swarm optimization method to solve the ED problem [4]. Gaing showed the result of proposed method is superior to genetic algorithm results. Park, Kim, and Jung used game theory to solve the ED problem [5]. The proposed model in [5] uses a strategy and decides the optimum economic dispatch strategy according to bidding prices.

In this paper, game theory with Nash equilibrium is applied by using Kurban and Basaran's [2] 14 bus power system to analyze the economic dispatch of real power generation. With this application, entire system is handled instead of Park, Kim, and Jung's application [5]. Nash equilibrium points are founded to decide the optimal operating points of generators.

The rest of the paper is organized as follows: Section II explains the economic dispatch problem. Section III defines the concept of game theory. Section IV presents a solution for a general economic dispatch problem using game theory. Section V shows MATLAB simulation results of the real example system. Conclusions are given in Section VI.

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Nezihe Yildiran is with Bahcesehir University, Energy Systems Engineering Department, Istanbul, Turkey (e-mail: nezihe.kucukyildiran@eng.bahcesehir.edu.tr).

Emin Tacer was with Bahcesehir University, Energy Systems Engineering Department, Istanbul, Turkey.

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II. Economic Dispatch Problem

Economic operation is very essential for a power system to return a profit on the capital investment. To determine the economic distribution of load between the various generating units, the variable operation costs of the plant must be expressed in terms of the power output. Fuel cost is the principal factor in fossil power plants. Analysis is based on the economics of fuel cost with the realization that other costs which are a function of power output can be included in the expression for fuel cost.

The factors influencing power generation at minimum cost are operating efficiencies of power generators, fuel cost, and transmission losses. As an example, the most efficient generator in the power system does not guarantee minimum total cost, because of it may be located in an area where fuel cost is high. Hence, the economic dispatch problem is to determine the generation of different plants while the total operating cost is minimum. The operating cost plays a key role in the economic scheduling.

The objective function of the economic dispatch problem is known as cost function. Cost functions consist of fuel cost, no load operating costs and initial operation costs of the generation plants. The aim of economic dispatch problem is minimizing the total cost of generation plants. Also, ED satisfies constraints such as total demand, generation limits, and transmission losses [6].

Generally, the input to the thermal power plants is measured in Btu/h (British thermal units per hour) and the output is measured in MW. An input-output curve of a thermal plant known as a heat-rate curve (Btu/h versus output power of the plant). Converting the ordinate of heat-rate curve from Btu/h to $/h presents the fuel-cost curve. Cost function of the generation plants gives the function of that fuel-cost curve. In all practical models, the cost function of i-th generation plant can be represented as a quadratic function of generated power and the formula is given by [6], [7].

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \tag{1}$$

System total cost function for n generator is defined as from (1),

$$C_T = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \tag{2}$$

The problem is to find the real power generation for each plant to make the total cost function minimum and supply the total load demand. Equality constraint of the system is given by

$$\sum_{i=1}^{n} P_i = P_D + P_L \tag{3}$$

The simplest economic dispatch problem is the scenario when transmission line losses are neglected. This problem does not consider system configuration and transmission line impedances. In theory, the model assumes that the system has only one bus with all generation plants and all loads connected to that bus.

When transmission distances are very small and load density is very high, transmission losses may be neglected and the economic dispatch of the generation plants is achieved with all plants operating at equal production cost. So, to simplify the system computations, transmission losses may be neglected. In this case, the system can be shown as in Fig. 1 and equality constraint will be

$$\sum_{i=1}^{n} P_i = P_D \tag{4}$$

For most economic operation, when losses are neglected with no generator limits, all plants must operate at equal production cost while satisfying the equality constraint.

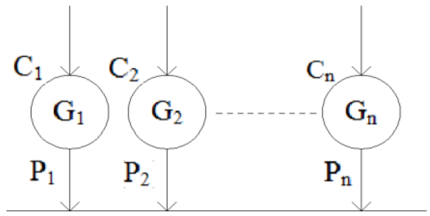

Fig. 1. System with n generators.

The power output of any generator should not exceed its operating ratings which are defines as the maximum and minimum generation amounts of the generator. So, the generator limits constraint (inequality constraint) is expressed as

$$P_{i(\min)} \le P_i \le P_{i(\max)} \quad i = 1,2,\dots,n \tag{5}$$

The problem is to find the real power generation for each plant such that the objective function as defined by (2) is minimum, subject to the equality constraint given by (4) and the inequality constraint given by (5).

III. Game Theory

Game theory is a discipline which helps to find out optimum choice. It is used firstly in economics to analyze different strategies. Today, game theory is applied to other branches of science. The fundamental insight of game theory was to apply the logic of games to events in real life.

A game-theoretic model is an environment where actions of each decision maker interact with other decision makers. Game theory uses economic and mathematical tools to solve decision making problems.

A game is a description of strategic interaction that includes the constraints on the actions that the players can take and the players' interests, but does not specify the actions that the players take. A solution is a systematic description of the outcomes that may emerge in a family of games. Game theory suggests reasonable solutions for classes of games and examine their properties.

Nash equilibrium is one of the most basic concepts in the game theory and the widespread method of predicting the outcome of a strategic decisions. First of all, Antoine Cournot used game theory and Nash equilibrium in 1838. The study was about duopoly. In Cournot's theory, firms decide their production amounts to maximize their own profit and it depends on the outputs of the others. During the 1950s, John Nash proved that finite games have always an equilibrium point. Equilibrium is defined as a stable outcome based on the payoffs received by players at the end of the game. Also, all players choose strategies which are best for them according to given their opponents' choices at this equilibrium point [8]. In Nash equilibrium, each player responds to the others with best decision.

A game includes three key ingredients: players, actions, and payoffs. The description of these three elements and their functions will be explained in the following example:

- A set of players $N = (\{1, 2, \dots, n\})$ is a finite set of n, indexed by i.
- A set of actions (pure strategies) available to each player $(a = (a_1, a_2, \dots, a_n) \in A)$ determines their possible moves or strategies.
- A payoff functions $\big(u = (u_1, u_2, \dots u_n)\big)$ represent each player's preferences and shows what players receive at the end of the game.

Best response in a game is defined as $a_i^* \in BR(a_{-i})$ if $\forall a_i \in A_i, u_i\big(a_i^*, a_{-i,}\big) \geq, u_i\big(a_i, a_{-i,}\big)$.

$a = (a_1, a_2, \dots, a_n)$ is a pure strategy Nash equilibrium if $\forall i, a_i \in BR(a_{-i})$.

On the other hand, the existence of an equilibrium is an important situation in game theory. The equilibrium point can be pure strategy or mixed strategy. When the pure strategy equilibrium point exists, equilibrium point shows maximum profit. If there is mixed strategy equilibrium point, the players or companies can choose maximum profit with a probability. Briefly, a pure strategy Nash equilibrium is a point which any player cannot gain a higher payoff deviating its profile alone [9].

Table I shows an example payoff matrix of the two player game. In this table, player 1 (P1) and player 2 (P2) have two strategies (strategy "A" and strategy "B"). The payoffs of these strategies are also given in the same table. If player 1 choose strategy "A", then player 2 will choose strategy "B" due to the higher payoff (1, 2). After that, player "A" changes his strategy to "B", but player 2 does not want to change his strategy in this case, must stay at strategy "B" (3, 3). As a result of this game, (B, B) strategy profile is the unique and pure Nash equilibrium point.

TABLE I: AN EXAMPLE OF TWO PLAYER GAME

P1 \ P2	A	B
A	(0,0)	(1,2)
B	(2,1)	(3,3)

IV. ECONOMIC DISPATCH PROBLEM USING GAME THEORY

Before the system analyze, game theory elements of the system should be clarified. In this paper, game theory elements with their power generation system equivalents are defined at Table II and flow chart of the proposed algorithm is given at Fig. 2.

TABLE II: GAME THEORY AND POWER SYSTEM EQUIVALENT

Game Theory Elements	Power System Equivalent
Players	Generation Plants
Strategies	Produced Power
Payoffs	1/0 According to Demand

To explain the algorithm, an example two generator simplified game can be defined and generator payoff matrix is given in Table III. According to the Table III, two players, generation plant 1 and generation plant 2 has been defined with two different production amounts; 100 MW or 200 MW options.

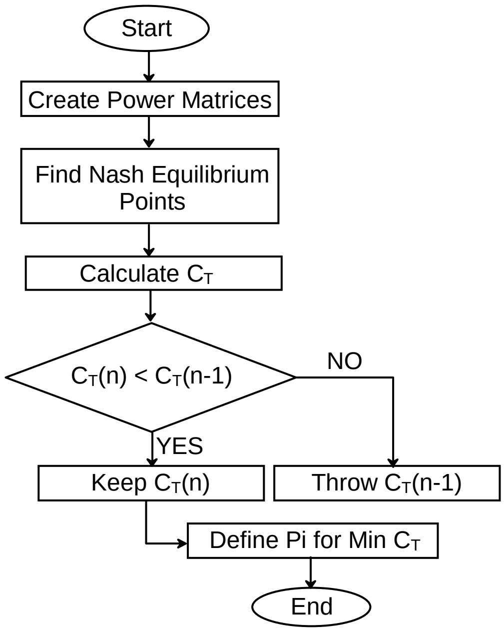

Fig. 2. Flow chart of proposed algorithm.

TABLE III: NASH EQUILIBRIUM POINTS OF TWO GENERATION PLANT

GP_1 \ GP_2	100 MW	200 MW
100 MW	(0,0)	(1,1)
200 MW	(1,1)	(0,0)

If the system total demand is equal to 300 MW, Nash equilibrium points, which supply the total demand are 100 MW, 200 MW and 200 MW, 100 MW. Nash equilibrium points are signed in Table IV.

TABLE IV: AN EXAMPLE OF TWO GENERATION PLANT

GP_1 \ GP_2	100 MW	200MW
100 MW	(0,0)	(1,1)
200 MW	(1,1)	(0,0)

After the loop of the presented algorithm, detected Nash equilibrium points are used to calculate total cost of the system. Minimum of calculated costs gives the system minimum total cost, in other words this point shows optimum operating strategy.

V. SIMULATION RESULTS

Fig. 3 shows 14 buses, 6 thermal generators power system in Turkey. The main objective of this paper is to solve this system using game theory Nash equilibrium. Also, Table V gives the generator coefficients and generator maximum and minimum values of 6 thermal power plants [2].

In this presented system, 6 thermal plants shows the players, the production amount of each generation plant shows the strategies, and the payoffs are determined according to the data at each point.

At the start of the game, all power possibilities of the generators were determined according to Table V system generator parameters. While the determination of the power possibilities, generator minimum and maximum power production amounts are taken into account and step size (power increase interval amount) are decided. As an example, for Bursa Natural Gas station the possible power production changes between 318MW and 1432MW, and step size is chosen as 0.0001MW. After that, power values of each power plant which satisfy the total demand amount, are found out. These points show also the Nash equilibrium points of the system. Total cost of the system is calculated for each Nash equilibrium point. Finally, minimum of the calculated total costs gives the system optimum operating point. The generator power amounts at founded point give the operation values of the generators and shows the optimum operation strategy.

TABLE V: GENERATOR PARAMETERS

Thermal Plants	$P_{i(min)}$	$P_{i(max)}$	α_i	β_i	γ_i
Bursa N. Gas (G1)	318	1432	6780.5	5.682	0.0106
Seyitömer (G2)	150	600	1564.4	3.1288	0.0139
SomaB (G3)	210	990	5134.1	6.2232	0.0168
Yeniköy (G4)	110	420	1159.5	3.3128	0.0210
Kemerköy (G5)	140	630	1697	3.2324	0.0137
Yatağan (G6)	140	630	1822.8	3.472	0.0147

At Table VI, the calculated total cost of the system with each generator production amount is given and compared with Lagrange Function (LF) [2] and genetic algorithm (GA) [3] solutions of the same system in the literature.

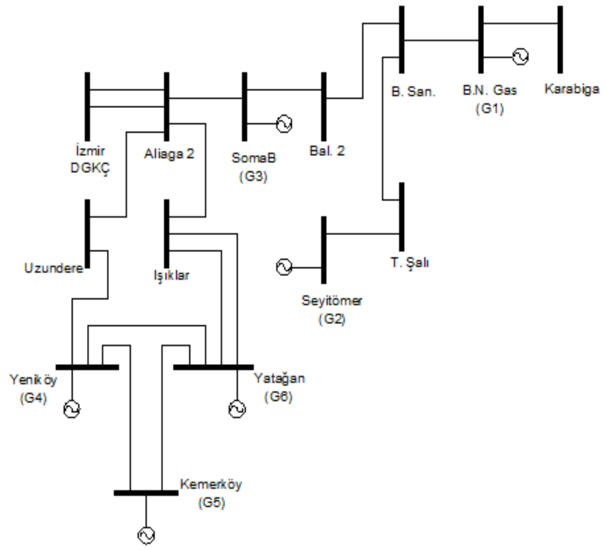

Fig. 3. 14 bus, 6 generator system [2].

Total demand is obtained by the generation plants with a calculation error. Error amount shows the difference between calculated demand amount and target demand amount. In game theory results, obtained total generator production amounts is 2735 MW while demand is 2734.9 MW. There is 0.1 MW calculation error. Comparison with other literature solutions shows that game theory error amount is approximately less than approximately 95% of others error amount.

Table VI shows clearly the minimum total cost is achieved by game theory solution with minimum error.

TABLE VI: COMPARISON OF THE RESULTS WITH ERROR (DEMAND=2734.9 MW)

Method	Error [MW]	Bursa N. Gas (G1) [MW]	Seyitömer (G2) [MW]	SomaB (G3) [MW]	Yeniköy (G4) [MW]	Kemerköy (G5) [MW]	Yatağan (G6) [MW]	Total Cost [$/h]
Lagrange Function [2]	42.7319	573.0010	520.3039	352.5975	335.5975	523.9189	472.2131	48.481,0000
Genetic Algorithm [3]	44.0000	552.0396	543.4736	322.6902	353.4248	515.1527	492.1534	48.454,9881
Game Theory	0.1000	555.0000	515.0000	334.0000	337.0000	519.0000	475.0000	47.662,6355

TABLE VII: COMPARISON OF THE RESULTS WITHOUT ERROR

Method	Demand [MW]	Bursa N. Gas (G1) [MW]	Seyitömer (G2) [MW]	SomaB (G3) [MW]	Yeniköy (G4) [MW]	Kemerköy (G5) [MW]	Yatağan (G6) [MW]	Total Cost [$/h]
Genetic Algorithm [3]	2734.9	554.0455	496.9588	320.8097	357.1712	519.2502	486.6204	47.679,2861
Game Theory	2734.9	554.9000	515.0000	334.0000	337.0000	519.0000	475.0000	40.313,9354

To get the exactly demand without error, error amount can be subtracted from a generation plant which has maximum cost coefficients to achieve lesser total cost. In this case, Generator 1 (Bursa N. Gas) has the maximum cost coefficients. So that, error amount was subtracted from Generator 1 to get the exact demand.

Table VII shows the generators operation amounts and minimum total cost without any error from demand. Moreover, total cost obtained from the game theory is 15.45% economy saving than total cost obtained from genetic algorithm.

VI. CONCLUSION

In this paper, solving economic dispatch problem using game theory Nash equilibrium has been presented. In the case study, the proposed method has been applied to 14 bus, 6 thermal generators of power system, which is selected from the Turkish utility power system.

The presented algorithm results show that the proposed game theory method gives the minimum total cost of the system with minimum error instead of Lagrange functions results and genetic algorithm results. Furthermore, if the computation is revised to find minimum total cost without error, total cost decrease. Finally, the results of this study shows that game theory helps to find better results for solving economic dispatch problem and to get economy saving up to 15.45 percent.

This power system may be studied with transmission losses or another power system with different types of generators including renewable energy resources may be analyzed for future work.

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Nezihe Yıldiran was born in Istanbul, Turkey, in 1980. She received the B.Sc. and M.Sc. degrees in electrical engineering in 2004 and 2007, respectively, from the Istanbul Technical University (ITU), Istanbul, Turkey. She is currently working toward the Ph.D. degree at Bahcesehir University, Istanbul, Turkey.

She has experience about power electronics research and development from the different companies between 2004 and 2012. She is a research assistant in the Energy Systems Engineering Department at Bahcesehir University since 2012. Her current research interests include modeling and control of power electronic converters, modeling renewable energy sources, and power electronic applications of smart grid.

Emin Tacer received Ph.D. degree from the Electrical Engineering Faculty of İstanbul Technical University (İTÜEE) in 1977. He did works as a postdoctoral research fellow at University of Toronto, Canada in 1978. In 1979, he was a visiting professor at University of Michigan. He became an associate professor in 1982, and a full professor in 1988, İTÜEE.

He has been emeritus since July 17, 2006 and had been a faculty member from 2006 to 2014 in the Engineering Faculty at Bahcesehir University and since 2015 has been a faculty member of the Engineering Faculty at Aydın University. He has supervised 5 Ph.D. dissertations and above 30 MSc. theses and he has been the advisor of 1 Ph.D. theses currently. He has published 5 transaction paper, 14 international articles, 3 books, 28 research reports and course notes. He gave consultancy for different company. He worked on different administrative positions at İTÜ and the other university. His academic fields and research areas are renewable energy sources, photovoltaic power generation, wind power generation, electrical machines, power electronics and electromagnetic.