Tourism, Growth and Business Cycles

Wei-Bin Zhang

***Abstract*—This study is concerned with business cycles in a small-open economic growth model with endogenous education and tourism. The model is based on Solow's one-sector and Uzawa's two-sector growth models, the Uzawa-Lucas two-sector growth model, some growth models for small open economies, the literature of growth and tourism. The model treats human capital growth, wealth accumulation and tourism as driving forces of growth. The small-open economy consists of one industrial sector, one service sector, and one education sector. The model integrates the main ideas in the literature by applying Zhang's concept of disposable income and utility function. This paper generalizes the model by allowing all time-independent coefficients to be time-dependent. The generalization makes it possible to examine impact of any time-dependent shocks such as seasonable tourism and random changes in interest rates in global changes.**

***Index Terms*—Business cycles, education, tourism, wealth accumulation.**

I. Introduction

This study is concerned with business cycles in the small-open economic growth with endogenous education and tourism proposed by [1]. Business cycles due to various exogenous shocks or endogenous forces are identified in the literature of economics [2]-[9]. But there are a few theoretical economic models with interactions between endogenous wealth, human capital, and tourism built on micro-economic foundation which show oscillations. The model by [t] is built on the basis of a few important models in economic theory. It is based on Solow's one-sector and Uzawa's two-sector growth models with capital accumulation as the machine of economic growth [10], [11], the Uzawa-Lucas two-sector growth model with education and human capital accumulation [12], [13], some growth models for small open economies [14], and the literature of growth and tourism [15]. It synthesizes various growth forces within a single comprehensive analytical framework. The model treats human capital growth, wealth accumulation and tourism as driving forces of growth. The small-open economy consists of one industrial sector, one service sector, and one education sector. The model integrates the main ideas in the literature by applying Zhang's concept of disposable income and utility function. This paper generalizes the model by allowing all time-independent coefficients to be time-dependent. The generalization makes it possible to examine impact of any time-dependent shocks such as seasonable tourism and random changes in interest rates in global changes. The paper is organized as follows. Section II generalizes Zhang's model small-open economic growth model with endogenous wealth, education and tourism. Section III shows a computational procedure for simulating movement of the economic system and simulates the model with constant coefficients. Section IV shows existence of business cycles due to periodic shocks. Section V concludes the study.

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II. The Growth Model with Education and Tourism

We are concerned with an open-small economy which freely trades with the rest of the world. The model is built on the main features of some well-known theoretical economic models. The capital accumulation and economic structure are especially influenced by the Solow-Uzawa growth model. The modelling of education and endogenous human capital is based on the Uzawa-Lucas two-sector growth model. We refer to [1] for explaining the basic structure. We now extend Zhang's model [1] by allowing all time-independent parameters to be time-dependent. Domestic households consume both goods and foreign tourists consume only services. Human capital and physical capital are depreciated at constant exponential rates. Domestic households own wealth and obtain income from wages and interest payments of wealth. There are capital and qualified labor force as input factors. All markets are perfectly competitive. Capital is perfectly mobile in international markets. There is a constant homogeneous population. We introduce following variables

i, s, e - subscript indexes for industrial, service, and education sectors;

T_0 - total available time for work, leisure, and education;

$K_j(t)$ and $N_j(t)$ - capital stocks and labor force employed by sector $j, j = i, s, e$;

$F_j(t)$ - output level of sector $j, j = i, s, e$;

$r^*(t)$ and $w(t)$ - exogenous rate of interest and wage rate;

$p(t)$ and $p_e(t)$ - prices of service and education;

$\bar{N}(t)$ and $N(t)$ - exogenous population and labor force;

$K(t)$ and $\bar{K}(t)$ - capital stocks employed and owned by the country, respectively;

$\bar{k}(t)$ - wealth per household; $\bar{K}(t) = \bar{k}(t)\bar{N}(t)$;

$T(t), \bar{T}(t)$ and $T_e(t)$ - work time, leisure time, and education time of the representative household;

$H(t)$ - level of human capital; and

$\delta_k(t)$ and $\delta_H(t)$ - exogenous depreciation rates of physical capital and human capital.

National labor force

National labor force is a function of human capital, labor time and total population. For simplicity of analysis, we specify the labor force function as follows:

$$N(t) = H^{m(t)}(t)T(t)\bar{N}(t), \tag{1}$$

in which $m(t)$ is positive and measures human capital utilization efficiency. A rise in the parameter implies that the

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Production sectors

The production function of industrial sector is taken on the following form

$$F_i(t) = A_i K_i^{\alpha_i(t)}(t) N_i^{\beta_i(t)}(t), \alpha_i(t), \beta_i(t) > 0, \alpha_i(t) + \beta_i(t) = 1, \quad (2)$$

where $A_i(t)$, $\alpha_i(t)$, and $\beta_i(t)$ are parameters. The total factor productivity and output elasticities of the two inputs are changeable. The firms are assumed to maximize the profit. The marginal conditions mean

$$r_\delta(t) = \alpha_i(t) A_i(t) k_i^{-\beta_i(t)}(t),$$
$$w(t) = \beta_i(t) A_i(t) k_i^{\alpha_i(t)}(t), \quad (3)$$

where $k_i(t) \equiv K_i(t)/N_i(t)$ and $r_\delta(t) \equiv r^*(t) + \delta_k(t)$.

The production function of service sector is

$$F_s(t) = A_s(t) K_s^{\alpha_s(t)}(t) N_s^{\beta_s(t)}(t), \alpha_s(t), \beta_s(t) > 0, \alpha_s(t) + \beta_s(t) = 1. \quad (4)$$

The marginal conditions imply

$$\frac{r_\delta(t)}{\alpha_s(t) A_s(t)} = p(t) k_s^{-\beta_s(t)}(t),$$
$$\frac{w(t)}{\beta_s(t) A_s(t)} = p(t) k_s^{\alpha_s(t)}(t), \quad (5)$$

where $k_s(t) \equiv K_s(t)/N_s(t)$.

The production function of education sector is

$$F_e(t) = A_e(t) K_s^{\alpha_e(t)}(t) N_e^{\beta_e(t)}(t), \alpha_e(t), \beta_e(t) > 0, \alpha_e(t) + \beta_e(t) = 1. \quad (6)$$

The marginal conditions are

$$\frac{r_\delta(t)}{\alpha_e(t) A_e(t)} = p_e(t) k_e^{-\beta_e(t)}(t),$$
$$\frac{w(t)}{\beta_e(t) A_e(t)} = p_e(t) k_e^{\alpha_e(t)}(t), \quad (7)$$

where $k_e(t) \equiv K_e(t)/N_e(t)$.

Full employment of capital and labor

We express full employment of labor force

$$N_i(t) + N_s(t) + N_e(t) = N(t). \quad (8)$$

We express full employment of physical capital

$$K_i(t) + K_s(t) + K_e(t) = K(t), \quad (9)$$

Tourism demand function

Let $y_f(t)$ to stand for incomes in foreign countries. Tourism demand function is assumed to be dependent on foreign income and domestic price of services. We specify the tourism demand function as follows:

$$D_T(t) = a(t) y_f^{\varphi(t)}(t) p^{-\varepsilon(t)}(t), \quad (10)$$

where $\varphi(t)$ and $\varepsilon(t)$ are the income and price elasticities of tourism demand. The variable, $a(t)$, is a parameter.

Domestic household behavior

Current income of household is

$$y(t) = r^*(t)\bar{k}(t) + H^m(t)T(t)w(t), \quad (11)$$

where $r^*\bar{k}(t)$ is the interest payment and $H^m(t)T(t)w$ the wage total payment. Disposable income is:

$$\hat{y}(t) = y(t) + \bar{k}(t). \quad (12)$$

The disposable income is spent on saving, education and consumption:

$$p_e(t)T_e(t) + p(t)c_s(t) + c_i(t) + s(t) = \hat{y}(t). \quad (13)$$

We have time constraint:

$$T(t) + \bar{T}(t) + T_e(t) = T_0, \quad (14)$$

Substitute (14) into (13)

$$H^{m(t)}(t)\bar{T}(t)w(t) + \bar{p}_e(t)T_e(t) + p(t)c_s(t) + c_i(t) + s(t) = \bar{y}(t), \quad (15)$$

in which

$$\bar{p}_e(t) \equiv p_e(t) + H^{m(t)}(t)w(t),$$

$$\bar{y}(t) \equiv \left(1 + r^*(t)\right)\bar{k}(t) + H^{m(t)}(t)T_0 w(t).$$

We interpret the above two variables the opportunity cost and the disposable income when all the time is devoted to work.

Utility function $U(t)$ is

$$U(t) = \theta(t)\bar{T}^{\sigma_0(t)}(t) T_e^{\eta_0(t)} c_s^{\gamma_0(t)}(t) c_i^{\xi_0(t)}(t) s^{\lambda_0(t)}(t),$$
$$\sigma_0(t), \eta_0(t), \gamma_0(t), \xi_0(t), \lambda_0(t) > 0,$$

where $\sigma_0(t)$, $\eta_0(t)$, $\gamma_0(t)$, $\xi_0(t)$, and $\lambda_0(t)$ are respectively called propensities to consume the leisure time, to receive education, to consume services, to consume industrial goods, and to hold wealth. Marginal conditions for maximizing $H_q^{\theta_q} T_{qe}$. subject to (15) imply:

$$\bar{T}(t) = \frac{\sigma(t)\bar{y}(t)}{H^{m(t)}(t)w(t)}, T_e(t) = \frac{\eta(t)\bar{y}(t)}{\bar{p}_e(t)},$$
$$c_s(t) = \frac{\gamma(t)\bar{y}(t)}{p(t)}, c_i(t) = \xi(t)\bar{y}(t), s(t) = \lambda(t)\bar{y}(t), \quad (16)$$

in which

$$\sigma(t) \equiv \rho(t)\sigma_0(t), \eta(t) \equiv \rho(t)\eta_0(t), \gamma(t) \equiv \rho(t)\gamma_0(t), \xi(t) \equiv \rho(t)\xi_0(t), \lambda(t) \equiv \rho(t)\lambda_0(t), \rho(t) \equiv \frac{1}{\sigma_0(t) + \eta_0(t) + \gamma_0(t) + \xi_0(t) + \lambda_0(t)}.$$

It should be noted that in our approach saving is given by s. Change in wealth is saving minus dissaving

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t). \quad (17)$$

Market equilibrium

The sum of domestic households' and tourists' demands for services is equal to the total supply. Service market equilibrium implies:

$$c_s(t)\bar{N} + D_T(t) = F_s(t). \quad (18)$$

The demand for education is equation to supply of education. Education market equilibrium implies

$$T_e(t)\bar{N} = F_e(t). \quad (19)$$

Accumulation of human capital

We consider that human capital change is due to the efficiency of efforts in education and depreciation of human capital. The efficiency is related to efforts of teaching, students' time and human capital, and returns to scale in education. We specify human capital accumulation equation as follows

$$\dot{H}(t) = \frac{\upsilon_e(t)\left(\frac{F_e(t)}{\bar{N}(t)}\right)^{a_e(t)}\left(H^{m(t)}(t)T_e(t)\right)^{b_e(t)}}{H^{\pi_e(t)}(t)} - \delta_h(t)H(t), \quad (20)$$

where $\upsilon_e(t)$, $a_e(t)$, $b_e(t)$ and $\pi_e(t)$ are parameters.

We completed building the model. Although it has many variables and these variables are related in nonlinear functions, we can explicitly demonstrate its movement over time with initial conditions.

III. The Dynamics of the Economy

We now show that the movement of the economic system is described by two differential equations.

Lemma

The variables, $k_i(t)$, $k_s(t)$, $w(t)$, $p(t)$, and $p_e(t)$ are uniquely determined as functions of t. Wealth and human capital are determined by the following two differential equations:

$$\dot{\bar{k}}(t) = \Omega_k\left(\bar{k}(t), H(t), t\right), \dot{H}(t) = \Omega_H\left(\bar{k}(t), H(t), t\right), \quad (21)$$

in which Ω_k and Ω_H are functions of $\bar{k}(t)$, $H(t)$ and t defined in the Appendix. We decide all the variables as functions of $\bar{k}(t)$, $H(t)$ and t as follows: $k_i(t)$ and $w(t)$ by (A1) $\rightarrow$ $k_s(t)$ by (A2) $\rightarrow$ $k_e(t)$ by (A3) $\rightarrow$ $p(t)$ by (5) $\rightarrow$ $p_e(t)$ by (7) $\rightarrow$ $\bar{y}(t)$ by (15) $\rightarrow$ $\bar{T}(t)$, $T_e(t)$, $c_i(t)$, $c_s(t)$, $s(t)$ by (16) $\rightarrow$ $D_T(t)$ by (10) $\rightarrow$ $N_e(t)$ by (A6) $\rightarrow$ $N_s(t)$ by (A7) $\rightarrow$ $N_e(t)$ by (A6) $\rightarrow$ $T(t)$ by (A8) $\rightarrow$ $N(t)$ by (1) $\rightarrow$ $N_i(t)$ by (A10) $\rightarrow$ $K_j(t) = k_j N_j(t)$ $\rightarrow$ $\hat{y}(t)$ by (12) $\rightarrow$ $F_i(t)$ by (2) $\rightarrow$ $F_s(t)$ by (4) $\rightarrow$ $F_e(t)$ by (6).

The Lemma enables us to follow the movement of the system with any change in any parameter(s). The reminder of this section summarizes the simulation result in [1] when the parameters are time-dependent. The result is the referring point for our following comparative dynamic analysis with different exogenous periodic shocks. We specify:

$D_T(t) = 0.5\bar{k}^{0.3}(t)y_f^{\varphi}p^{-\varepsilon}$, $\quad r^* = 0.06, \delta_k = 0.05, \bar{N} = 10, T_0 = 24,$
$L = 20, A_i = 1.2, A_s = 1, \alpha_i = 0.3, \alpha_s = 0.31, \alpha_e = 0.35,$
$\beta_s = 0.6, \lambda_0 = 0.6, \xi_0 = 0.15, \gamma_0 = 0.06, \eta_0 = 0.02,$
$\sigma_0 = 0.2, a = 1, y_f = 4, \varphi = 1.5, \varepsilon = 1.5, m = 0.6,$
$\delta_h = 0.04, \upsilon_e = 0.7, a_e = 0.2, b_e = 0.4, \pi_e = 0.3.$ (22)

In order to follow the movement of the dynamic system, we need to specify initial conditions. We plot the movement of the system with Figure 1 with the following initial conditions:

$$\bar{k}(0) = 255, H(0) = 15.$$

The dynamics is plotted in Fig. 1. In Fig. 1, we define $Y(t) \equiv F_i(t) + p_s F_s(t) + p_e F_e(t)$.

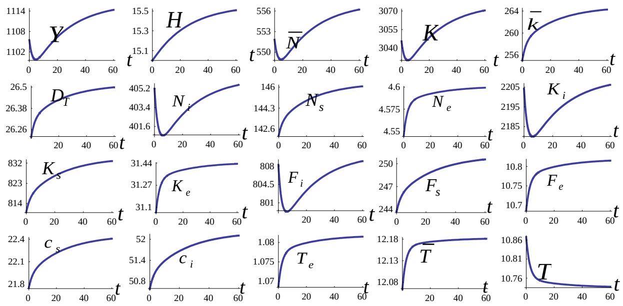

Fig. 1. The motion of the national economy.

Fig. 1 shows convergence of the dynamic system in the long term. It is straightforward to identify the equilibrium values as follows:

$$w = 1.4, p = 1.18, p_e = 0.91, Y = 1116.4, H = 15.6,$$
$$N = 557.2, K = 3076.1, \bar{K} = 2648.7, E = -25.6,$$
$$DT = 26.5, N_i = 406.3, N_s = 146.4, N_e = 4.6, K_i = 2210.1,$$
$$K_s = 834.6, K_e = 31.5, F_i = 810.4, F_s = 251.1, F_e = 10.8,$$
$$\bar{k} = 264.9, c_i = 52.2, c_s = 22.5, T_e = 1.08, \bar{T} = 12.18,$$
$$T = 10.74.$$

It is straightforward to calculate the three eigenvalues at the equilibrium point as follows:

$$\{-0.393, -0.04\}.$$

We thus see that the equilibrium point is locally stable. This result is very important for our analysis as it guarantees the validity of comparative dynamic analysis with regards to various exogenous shocks in the next section.

IV. Comparative Dynamic Analysis

The previous section depicts the movement of the system when all the parameters are constant as specified in Zhang (2017). This section demonstrates business cycles when some parameters are subject to periodic shocks. We introduce a variable, $\bar{\Delta}x(t)$, to present the change rate of the variable, $x(t)$, in percentage due to changes in the parameter value.

The propensity to receive education periodically oscillates

First, we consider a case when the propensity to received education is subject to the following exogenous

$$\eta_0(t) = 0.02 + 0.022\,sin(t).$$

We first observe that the time-independent variables are not affected by the preference change, i.e, $\bar{\Delta}p = \bar{\Delta}p_e = \bar{\Delta}w = 0$. We plot the rest of the simulation results in Fig. 2. We see that all the variables in Fig. 2 oscillates around the corresponding variables in Fig. 1.

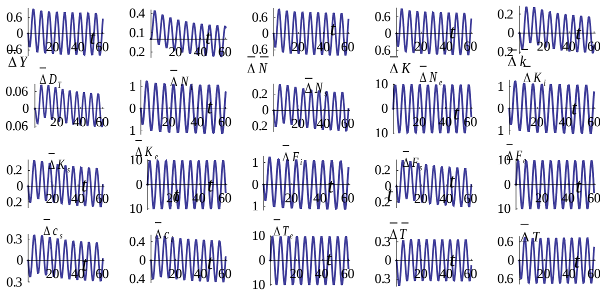

Fig. 2. The propensity to receive education oscillates.

The propensity to enjoy leisure time periodically oscillates

We are now concerned with the effects on the economy due to the following rise in the propensity to enjoy leisure time:

$$\sigma_0(t) = 0.2 + 0.22\, sin(t).$$

The time-independent variables are not affected by the preference change. We plot the simulation result with Fig. 3.

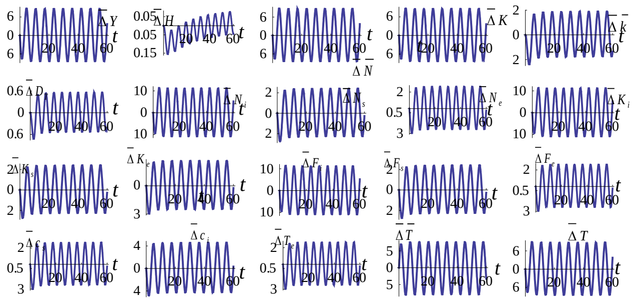

Fig. 3. The propensity to enjoy leisure time periodically oscillates.

V. Conclusions

This study reveals existence business cycles in the small-open economic growth with endogenous education and tourism proposed by [1]. Zhang's model is based on Solow's one-sector and Uzawa's two-sector growth models, the Uzawa-Lucas two-sector growth model, some growth models for small open economies, the literature of growth and tourism. The model integrates the main ideas in the literature by applying Zhang's concept of disposable income and utility function. This paper generalized the model by allowing all time-independent coefficients to be time-dependent. We examined the effects of time-dependent periodic shocks on the economic system. We only varied two parameters in demonstrating existence of exogenous business cycles. It is obvious that it is straightforward to vary any parameter in the system. It is also interesting to see how the system behaves when there are changes simultaneously in multiple parameters. As the model is founded on some well-known models and each of these famous models has generated a large literature, it is not difficult for us to extend and generalize our model on the basis of the extensive literature. We can also apply other forms of utility and production functions. It is important to allow domestic households to be tourists.

Appendix: Proving the Lemma

From (3) we express the capital intensity of the industrial sector and wage rate as a function of the rate of interest and the total factor productivity

$$k_i = \left(\frac{\alpha_i A_i}{r_\delta}\right)^{1/\beta_i}, w = \beta_i A_i k_i^{\alpha_i}. \tag{A1}$$

From (5) we solve the capital intensity of the service sector as a function of the rate of interest

$$k_s = \frac{\alpha_s w}{\beta_s r_\delta}. \tag{A2}$$

From (7) we solve the capital intensity of the education sector as a function of the rate of interest

$$k_e = \frac{\alpha_e w}{\beta_e r_\delta}. \tag{A3}$$

Equation (9) is rewritten as

$$k_i N_i + k_s N_s + k_e N_e = K. \tag{A4}$$

From (16) and (19) we obtain

$$\frac{\eta \bar{y} \bar{N}}{\bar{p}_e} = F_e. \tag{A5}$$

Substituting (6) into (A5), we get the labor force employed by the education sector as follow:

$$N_e = \frac{\eta \bar{y} \bar{N}}{A_e k_e^{\alpha_e} \bar{p}_e}. \tag{A6}$$

We solve (16), (18) and (4) with the labor force employed by the service sector as the variable:

$$N_s = \left(\frac{\gamma \bar{y} \bar{N}}{p} + D_T\right) \frac{1}{A_s k_s^{\alpha_s}}. \tag{A7}$$

Equations (16) and (14) imply:

$$T = T_0 - \frac{\sigma \bar{y}}{H^m w} - \frac{\eta \bar{y}}{\bar{p}_e}. \tag{A8}$$

From (A8) and (1) we solve the total labor force of the national economy

$$N = \left(T_0 - \frac{\sigma \bar{y}}{H^m w} - \frac{\eta \bar{y}}{\bar{p}_e}\right) H^m \bar{N}. \tag{A9}$$

With (8) we solve the labor force employed by the capital goods sector as follows:

$$N_i = N - N_s - N_e. \tag{A10}$$

We thus have the values of the variables as functions of $\bar{k}$ and H, and t as described in the Lemma. From the procedure in the Lemma, (17) and (20), it can be seen that we get the different equations for wealth and human capital, respectively, as follows:

$$\dot{\bar{k}} = \Omega_k(\bar{k}, H, t) \equiv s - \bar{k},$$
$$\dot{H} = \Omega_H(\bar{k}, H, t) \equiv \frac{\upsilon_e \left(\frac{F_e}{\bar{N}}\right)^{a_e} (H^m T_e)^{b_e}}{H^{\pi_e}} - \delta_h \tag{A11}$$

We did not provide explicit expressions of the functions as they are too tedious. Nevertheless, in simulation computer calculates the variable variables easily by following the computational procedure. We thus proved the Lemma.

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REFERENCES

[1] W. B. Zhang, "Education and Tourism in a small open growth economy," *International Journal of Academic Research in Business and Social Sciences*, vol. 7, no. 11, pp. 643-61, 2017.

[2] W. B. Zhang, *Synergetic Economics*, Heidelberg: Springer-Verlag, 1991.

[3] W. B. Zhang, *Differential Equations, Bifurcations, and Chaos in Economics*, Singapore: World Scientific, 2005.

[4] W. B. Zhang, *Discrete Dynamical Systems, Bifurcations and Chaos in Economics*, Elsevier: Amsterdam, 2006.

[5] H. W. Lorenz, *Nonlinear Dynamic Economics and Chaotic Motion*, Berlin: Springer-Verlag, 1993.

[6] C. Chiarella and P. Flaschel, *The Dynamics of Keynesian Monetary Growth: Macro Foundations*, Cambridge: Cambridge University Press, 2000.

[7] R. Shone, *Economic Dynamics – Phase Diagrams and Their Economic Application*, Cambridge: Cambridge University Press, 2002.

[8] G. Gandolfo, *Economic Dynamics*, Berlin: Springer, 2005.

[9] T. Puu, *Nonlinear Economic Dynamics*, Berlin: Springer, 2011.

[10] R. Solow, "A contribution to the theory of growth," *Quarterly Journal of Economics*, vol. 70, no. 1, pp. 65-94, 1956.

[11] H. Uzawa, "On a two-sector model of economic growth," *Review of Economic Studies*, vol. 29, no. 1, pp. 47-70, 1961.

[12] H. Uzawa, "Optimal technical change in an aggregative model of economic growth," *International Economic Review*, vol. 6, no.1, pp. 18-31, 1965.

[13] R. E. Lucas, "On the mechanics of economic development," *Journal of Monetary Economics*, vol. 22, no. 1, pp. 3-42, 1988.

[14] M. Obstfeld and K. Rogoff, *Foundations of International Macroeconomics*, Mass., Cambridge: MIT Press, 1996.

[15] D. Z. Zeng and X. W. Zhu, "Tourism and industrial agglomeration," *The Japanese Economic Review*, vol, 62, no. 4, pp. 537-61, 2011.

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