Effect of Behavioral Finance on Gold Price Trend

L. Dieupart-Ruel, A. François, A. Grauffel, J. Le Roy, and T. Vacheret

***Abstract*—Since the 60's, the Efficient Market Hypothesis (EMH) is under question. From recent events, particularly related to the 2008 Financial Crisis, financial assets appear as uncorrelated with their fundamentals and their prices are subject to strong fluctuations. In this context, including behavioral finance in financial system dynamics could be a response. On a more limited scope, the goal of present project is to establish a modeling process of gold price trend based on 1980 Grossman-Stiglitz model by integrating behavioral component for a better representation of markets reality.**

The modeling process is based on simulations and the article presents selected cognitive biases. The analysis of the results, and the corrections that followed, lead to the formulation of a mathematical model. Thus, the impact of cognitive biases on dynamics of gold price formation is demonstrated, and evaluated through their influence on informed (rational and irrational) and uninformed investors. The result of this study suggests that behavioral finance has a crucial part in gold price formation.

***Index Terms*—Behavioral finance, biases, gold, multi-agent based modeling.**

I. Introduction

Since E. Fama, financial theory rests upon Markets Efficiency Hypothesis (MEH) which supposes that financial assets prices equal their fundamental value, ie that there is no difference between price and value as a consequence of economical agents rationality and/or arbitrages correcting possible price anomaly. According to [1] definition: ‘*’on efficient market, asset price is a good indicator of its intrinsic value*’’, and ‘*’on efficient market, competition will manage in such a way that in the average all new information on intrinsic asset value will be instantaneously reflected in the price’’*, considered as evidences in his time. On Dec. 1996, during Internet bubble, director of FED A. Greenspan alluded to “irrational exuberance” in an official talk. By irrationality is meant excessive active assets price volatility, markets non efficiency, financial bubbles and other panic phenomena. Director of IFM C. Lagarde did also mention “total markets irrationality” just after 2008 world financial crisis. So despite its past success, E. Fama hypothesis is questionable. Faced with enormous changes, excessive investors reaction results in under/over financial assets estimation. There is today a common agreement on psychological influences exerted on economic agents, leading to financial assets price disconnection from “real” value.

In this context, behavioral finance could provide an element of response to the split of assets value. Defined as the study of human behavior and its consequences on investment decisions and price formation, behavioral finance may possibly explain asset price anomaly and is opposing to usual MEH.

Most studies in this broad research field are mainly devoted to social aspects, with very few mathematical models. Most of them are based on multi-agent Grossman-Stiglitz approach [2] with different objectives, agent choice, biases and considered assets.

From observation of MEH inadequacy and markets irrationality, present study is intended to provide an explanation to existing disconnection between financial asset price and its intrinsic value by analyzing how behavioral finance plays a role in price formation. The goal is to evaluate the impact of individual and collective biases on market irrationality and inefficiency through modeling behavioral component. To make the problem a well defined one, present analysis will focus on specific gold market, and will take advantage of market quantitative analysis tools to single out the proper role of behavioral finance. So after model presentation in a first part, information and cognitive biases will be discussed in a second part. Equilibrium prices under combined actions will be derived in the last part.

II. Model Presentation

The model used here extends [2] without HEM/HAR paradigm. Perfect concurrence is supposed between informed and non-informed investors, market is of Walrassian type implying arbitrage and a commissioner with equilibrium price equating offer and demand. The model, inspired from Kaestner [3]-[4], is restricted to gold trade. This is a limited market where industry and jewelry represent 70% buyer share, the remaining 30% corresponding to speculators and hoarders, so that price variations can be quite large. Gold price is sensitive to financial stability and to decisions of Central Banks which are holding about 50 times gold annual production in their safes. As gold is traded in US dollar currency at NYMEX there can be a change risk. Let P the gold equilibrium price and v its true value supposed to follow a normal law with mean value μ and variance σ_v^2 ie $v = \mu + \varepsilon_v{*}$ with $\varepsilon_v{*} \approx N(0,\sigma_v)$ normally distributed with mean value 0 and variance σ_v^2. Economic agents are participating to price formation from their own perception of gold value, strongly depending on available information out of which they generate their predictions. Nowadays where communication systems are so efficient, psychological influences cannot be neglected and are also contributing to this information. So three classes of investors will be defined:

1) Rational Informed Agents (RIA) who take into account the fundamentals, and receive a noisy signal on future gold value.
2) Irrational Informed Agents (IIA) who are moreover following cognitive biases, and perceive a noisy and biased signal on future gold value.

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3) Non Informed Agents (NIA) who are anticipating gold price from market observation. They do not receive any signal and follow market trend resulting from action of the other agents. They are influenced by price estimates and by behavioral biases of informed agents.

Let π_i π_b π_n the three classes of agents respectively, with $\pi_i + \pi_b + \pi_n = 1$. Let also W_n the final richness of investor n supposed to maximize his utility function according to received information $U(W_n) = -\exp(-a_n W_n)$.

III. Information Bias and Signal Properties

As indicated received information is not only made of regular rational knowledge from technical data but also include cognitive biases from psychological and subjective beliefs. Two types will be distinguished which will affect decisions of agents belonging to second class.

A. *Anchoring Biases*

Anchoring biases leading to fix key value as a base with a confidence centered interval. As a hypothesis it will be supposed that investors under-react to analysts' predictions because of existing anchoring bias. Different situations are leading to investor position change when gold real value stays:

1) After two consecutive periods
   - Outside the same limit, then investor confidence decreases and confidence interval reduces by 1%
   - Outside with a different limit, then nothing happens
   - Inside the limits, then investor confidence increases and confidence interval widens by 2%
2) After three consecutive periods
   - Outside the limits, then investor is no more confident in his own anchoring value. His new value will be based on the average of the true value during preceding three periods, and his confidence interval will be modified to $[\nu_a - 4\%, \nu_a + 4\%]$
   - Inside the limits, then investor confidence increases by the same amount 2% and so on after each consecutive period in the interval

Here confidence interval is bounded in between minimum value 2% and maximum one 10% and $\theta_a = \delta.(\nu_a - \nu)$, where θ_a is the anchoring bias, δ the sensitivity to bias with 20% < δ < 100%, ν_a the anchoring value and ν the true gold value.

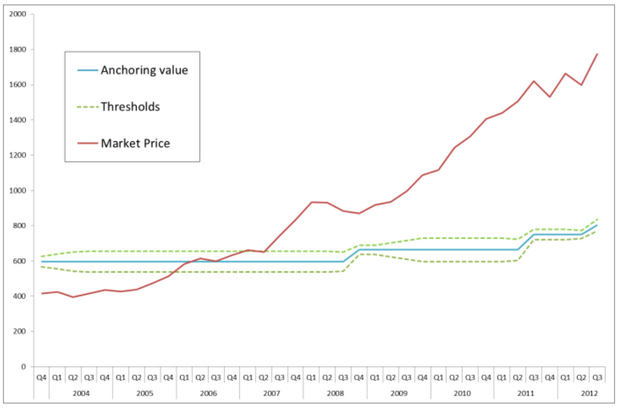

Fig. 1. Anchoring value and thresholds vs. time from last 2003 trimester compared to gold value

Biased Agent receives the time dependent price signal

$$s_t = \nu_t + \delta(\nu_a - \nu_t)$$

The anchoring value will be determined as a mean value of gold evaluated prices over the last three periods including the current one. Price revision will take place according to fixed thresholds $\theta_{s\pm} = \nu_a(1 \pm \delta_a)$ where δ_a is the confidence coefficient in actual value and ν_a fixes the center of confidence interval, see Fig. 1.

B. *Safe Value Biases*

Safe value bias is specific to gold which provide highest security in troubled periods with dark future. The bias represents investors' judgment against uncertainty. As observed, gold demand significantly correlates with markets instability, and one can write $\theta_r = \lambda\ \nu$, where θ_r is the safe value bias, λ an amplifying factor and ν the true gold value. This bias should be independently defined as gold evolution is anti-correlated to other assets such as money. It refers to agent belief that gold is safer than money in crisis period for instance. A description of this bias should be based on volatility indicator representing investors stress feeling. Under strong volatility, biased agent is overvaluing gold and conversely he undervalues gold in weak volatility periods.

The indicator used here is the VIX, calculated from the average of calls and puts on S&P500. The safe value bias will be evaluated as $s_t = \nu_t(1 + \Lambda)$, where ν_t is actual asset value at time t and Λ a value related to a fixed threshold value of VIX. Biased signal is built from Λ value defined from VIX value over preceding period, and modified by increase or decrease according to fixed volatility thresholds giving another time variation, see Fig. 2.

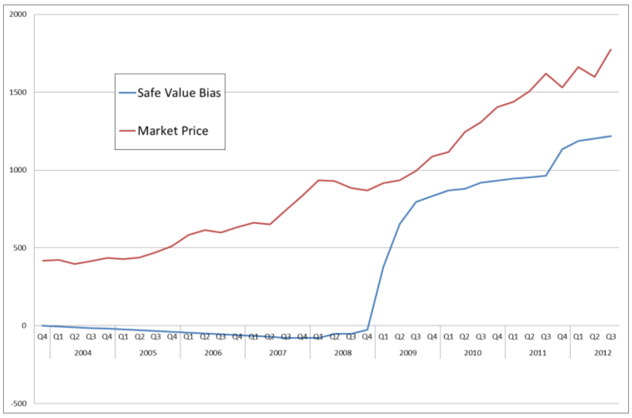

Fig. 2. Safe value bias for gold and market gold value vs. time

As indicated above the received signal is depending on agent class. Informed Agents (class 1 and 2) receive "price" information as an identical signal $s = \nu + \varepsilon_i^*$, with ν the true gold value and $\varepsilon_i^* \approx N(0,\sigma_\nu)$ the normal error term, whereas non informed agents will only receive equilibrium price P. Signal s is differently interpreted by the different agents. Rational ones (class 1) will determine the expectable gold value from signal s ie:

$$E\left(v^* / s\right) = \mu + \frac{cov\left(v^* s^*\right)}{v\left(s^*\right)}\left(s - \mu\right) \tag{1}$$

which represents a mean between information and signal weighted by a precision term $\tau_I = \mathrm{cov}(\nu^*, s^*)/V(s^*)$. For irrational agents (class 2) their deformed received signal writes $s = \nu + \varepsilon_b$ with $\varepsilon_b \approx N(\theta_b, \sigma_b)$. Supposing mutual independence of variables ν, ε_i, ε_b, the expected gold value is:

$$E_b\left(\nu / s\right) = \mu + \theta_b + \tau_b\left(s - \mu - \theta_b\right) \quad (2)$$

with τ_b the relative signal preciseness for biased agents.

IV. Equilibrium Price

The transaction price P is fixed by exact balance between offered and demanded prices. Second one is resulting from agents' maximization of their utility function. According to its property in [5], agent's demand writes

$$X = \frac{E\left(\nu / s\right) - P}{aV\left(\nu / s\right)} \quad (3)$$

where a is a risk aversion coefficient. Offer is obtained from data collected on gold market since 2003 on trimester basis, and is denoted by x. Let X_i the demand of class 1 agents (in percentage π_i), X_{bj} the demand of class 2 agents (j=1,2 for the two biases discussed in Part III), with respective percentage π_{bj} ($\pi_{b1} + \pi_{b2} = \pi_b$), and X_n the demand of third class agents (in percentage π_n) then equilibrium price P is determined by

$P = (p.E)/(p.1)$ where $p = [p_1, p_2, p_3, p_4]^T$, $E = [E_1, E_2, E_3, E_4]^T$, $1 = [1,1,1,1]^T$, with index 1,2,3,4 respectively corresponding to $1, b_1, b_2$, and u, and definition $p_k = \pi_k / a_k V_k(\nu|s)$. It will be supposed that there is a preliminary "practice" period during which class 3 agents are making up their price expectation value by observation of price expectation value $E_i(\nu|s)$ of agents belonging to the two other classes. So class 3 agents' information takes the form

$$\omega = E_i\left(\nu / s\right) + \lambda_{1,1}\Delta E_{bi,1} + \lambda_{1,2}\Delta E_{bi,2}\left(\nu / s\right) - \lambda_2 x \quad (4)$$

where $\Delta E_{bi,j} = E_{b,j}(\nu|s) - E_i(\nu|s)$, ($j = 1,2$), are estimation errors related to biases, and $\lambda_{1,k} = \pi_i a_i \gamma_k / D(\nu|s)$,

$\lambda_2 = a_b a_i V_i(\nu|s),/D(\nu|s)$, with $\gamma_k = V_i(\nu|s)/V_{bk}(\nu|s)$. Then their gold expectation value is given by

$$E\left(\omega\right) = \mu + \lambda_{1,1}\theta_{b,1}\left(1 - \tau_{b1}\right) + \lambda_{1,2}\theta_{b,2} - \lambda_2 x \quad (5)$$

and equilibrium price which will be used in the application finally becomes $P = (\mathbf{p}.\hat{\mathbf{E}})/(\mathbf{p}.\mathbf{1})$ where $\hat{\mathbf{E}} = [\omega, \omega, \omega, E(\nu|s)]^T$

V. Applications

To test model coherence, four different cases have been considered corresponding to different types of markets with different agents and different weightings. First case corresponds to 100% first class agents, second case to 100% second class agents with first bias effect, third case to 100% second class agents with second bias effect, and fourth case to a blend of all agents. The intention was to fix bounds on possible model limits against real market data. Results have been optimized by least square regression method.

Equilibrium prices obtained for each following case take into account the value called "fundamental price" of gold (calculated from global gold demand [6]) which is then impacted by various biases suffered by all three agents. This value is represented by the curve labeled "Calculated Price" on the figures below. The aim of our model is to be close to gold "Market Price" with our "Equilibrium Price", demonstrating so the effect of behavioral finance on gold price trend.

A. 100% First Class Agents

These agents are basing their price estimate on gold fundamental value. On the other hand an upward trend is observed on gold market usually explained by increasing demand, but this does not reflect market reality.

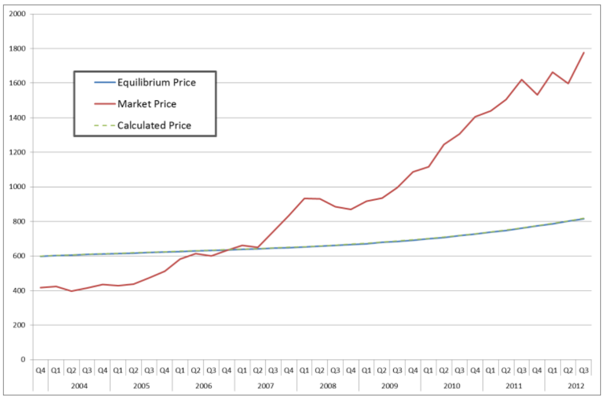

Fig. 3. Equilibrium, calculated and market prices vs. time

B. 100% Second Class Agents with Safe Value Bias

The obtained value follows market price trend but is over-evaluated and so, result does not fit market behavior. Participation of other agents has to be included for improving market description.

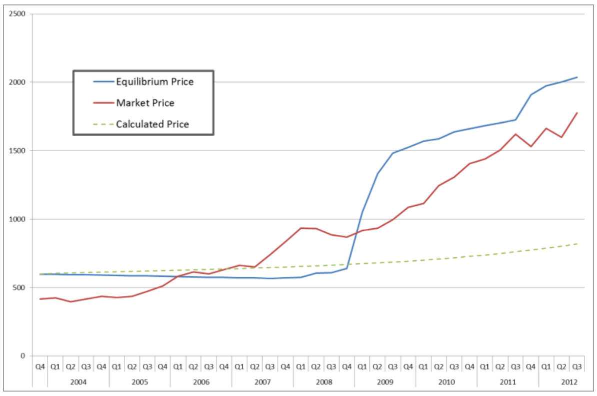

Fig. 4. Equilibrium, calculated and market prices vs. time

C. 100% Second Class Agents with Anchoring Value Bias

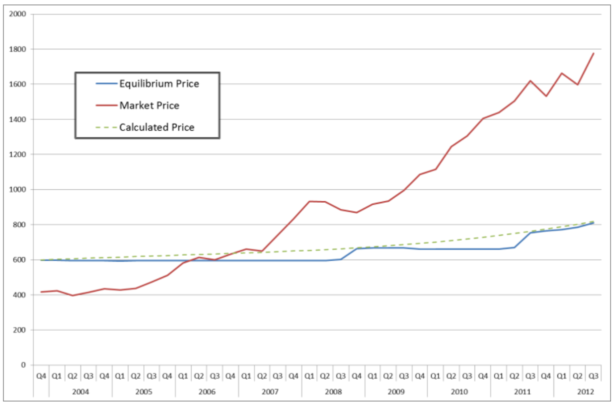

Fig. 5. Equilibrium, calculated and market prices vs. time

Results are similar to case 1, and obtained values do not reflect all highs and lows due to threshold effects resulting from anchoring process.

D. Optimum Agent Blend Fitting Actual Gold Price Evolution

From observations, proportion of informed agents (class 1 and 2) and uninformed ones (class 3) will be respectively taken as 82% and 18%. Even if agents belonging to last class are more important, this proportion weights correctly their influence in sales exchange. Amongst all informed agents, class 1 will represent 12.2%, and class 2 splits into 14.63% under anchoring bias and 73.17% under safe value bias. The last figure is not surprising owing to gold importance during economic, financial and political crisis. With previous fixed proportions, different periods can be distinguished in the analysis:

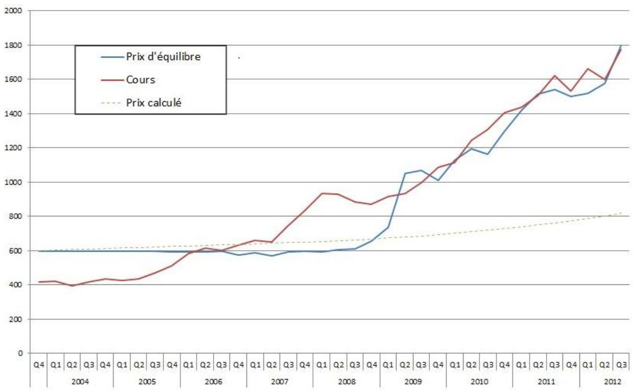

Fig. 6. Equilibrium, Calculated and Market Prices vs. Time

1) 2004-2008: volatility is weak, the effect of safe value bias is weak, amplifying coefficient is small. Equilibrium gold price is almost exactly equal to calculated price. Market price is following its own slowly growing trend and, in particular, is anticipating crisis disorder by a faster increase in second semester of 2007.
2) 2008-today: volatility is high due to subprime crisis, and amplifying coefficient of safe value bias is much higher, strongly influencing agents decision, making gold equilibrium price to jump (with one year delay) to market price. This last fact is not surprising owing to gold importance during economic, financial and political crisis. The observed oscillations of equilibrium gold price on all Figures and especially Figure 6 is essentially due to the scarcity of available data giving the intermediate evaluations relatively large fluctuations.

VI. Conclusion

Markets Efficiency Hypothesis (MEH) has been revisited and extension of Grossman-Stiglitz model has been proposed to explain the observed discrepancy between actual and expectable commodity value, with specific application to gold. Three classes of agents playing on the market have been identified according to their information and rationality levels:

1) Rational Informed Agents (RIA) taking into account the fundamentals, and receiving a noisy signal on future gold value,
2) Irrational Informed Agents (IIA) moreover following cognitive biases, and perceiving a noisy and biased signal on future gold value,
3) Non Informed Agents (NIA) anticipating gold price from market observation. They do not receive any signal and follow market trend resulting from action of the other agents. They are influenced by price estimates and by behavioral biases of informed class 2 agents. The biases they follow are identified as anchoring and safe value biases leading respectively to fix a reference value with uncertainty interval and to over/under valuate importance of market uncertainty.

Having established fundamental gold price based on gold demand, cognitive biases affecting irrational agents have been modeled in order to evaluate gold equilibrium price regulating the market. Application has been made to different possible situations to verify the impact of compensation of incomplete knowledge/non perfect rationality by biased decision. From the results, it can be verified that when applied to a recent past period equilibrium gold price is close to historical one. So adding behavioral components into the formation of fundamental price better approximates actual market gold price depending on the considered period and shows their importance from difference in the results corresponding to the different considered cases. The results also suggest that calculated gold price under and over-evaluation could be improved by more refined bias and indicators analysis in the model. Among the indicators that can explain the formation of the gold price, central banks' action is part of the answer. Acquisitions of gold by central banks have never stopped increasing during the last decade driving up the gold price on an ongoing basis for a twelfth consecutive year in 2012. If this indicator couldn't be modeled in this study, its integration will be subjected to our next publication.

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